



STATISTICAL PROCESS CONTROL

Part 2: How and Why SPC Works

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Part 1 in this series introduced Statistical Process Control (SPC) by discussing the history, philosophy, and benefits of SPC, by suggesting how to successfully implement a new SPC program, and by attempting to alleviate fears of the math associated with SPC.

The goal for Part 2 is to build management understanding and confidence in SPC as a profit-making tool. It is unreasonable to expect managers to commit to and support SPC training and implementation if they do not understand what SPC is and how and why it works. This publication discusses the importance of understanding and quantifying process variation and describes how and why SPC works.

Part 3 in the series is a step-by-step approach to building the skills required to implement SPC. Later publications in the series will present case histories of SPC use in wood products firms, examining pitfalls and successful approaches and providing real-world evidence of SPC's benefits as a process improvement tool.

Variation—it's everywhere

Variation is a fact of life. It is everywhere, and it is unavoidable. Even a brand-new, state-of-the-art machine cannot hold perfectly to the target setting; there always is some fluctuation around the target. Attaining consistent product quality requires understanding, monitoring, and controlling variation. Attaining optimal product quality requires a never-ending commitment to reducing variation.

Where does variation come from? Walter Shewhart, the man whose work laid the foundations for SPC, recognized that variation has two broad causes: *common* (also called *chance*, *random*, or *unknown*) causes and *special* (also called *assignable*) causes.

Common causes of variation are inherent in the process and can be thought of as the "natural rhythm of the process." Common causes are

evidenced by a stable, repeating pattern of variation. Real quality improvement requires a continual focus on reducing common-cause variation.

Special causes of variation are a signal that something has changed in the process. Special causes are evidenced by a disruption of the stable, repeating pattern of variation. Special causes of variation result in unpredictable process performance and must therefore be identified and removed before taking other steps to improve quality.

Why is it important to distinguish between these two types of variation? Because the remedies are completely different. Understanding the difference between the two types of variation helps manufacturers to target quality improvement efforts correctly and thereby avoid wasted effort and expense.

SPC in a nutshell

Montgomery (1997) defines SPC as “...a powerful collection of problem-solving tools useful in achieving process stability and improving capability through the reduction of variability.” Control charts and process capability analysis are the two primary tools of SPC. Other tools such as histograms, flow charts, cause-and-effect diagrams, check sheets, and Pareto diagrams also are useful in quality and process improvement.

In discussing how and why SPC works, we will:

- Describe the distribution of the process
- Estimate the limits within which the process operates under “normal” conditions
- Determine whether the process is stable
- Continue to monitor and control the process
- Compare process performance to specifications, and
- See how to continuously improve the process

The distribution of the process

In SPC, when we talk about the *distribution of the process*, we are referring not to the process itself but to data collected **from** the process—for example, data on widths of pieces coming out of a woodworking machine—and to the way those data are *distributed* when plotted on a chart or graph.

Describing the distribution of a process is analogous to evaluating your marksmanship. How’s your aim? Are you accurate—in other words, are you on target? Is your aim precise; that is, are all

Real quality improvement ...

requires a continual focus on reducing common-cause variation.

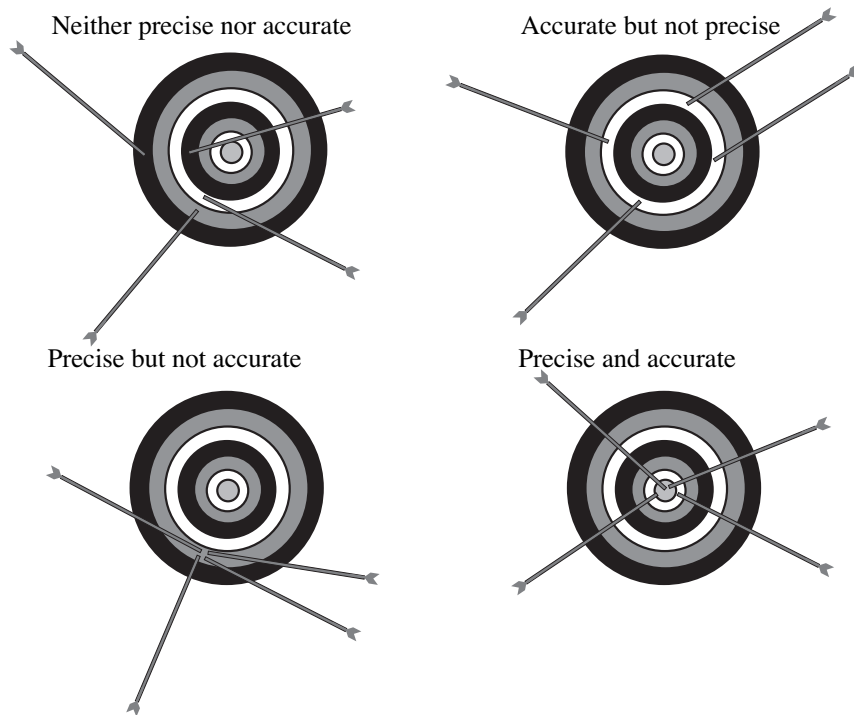


Figure 1.—Precision and accuracy (adapted from Montgomery, 1997).

the shots clustered around the target, or are they distributed all over the place (Figure 1)? In manufacturing, the questions are:

- Where is the process *centered*; in other words, is the process on or off target and, if it's the latter, by how much is it off target?
- How much does the process fluctuate about the center?

Histograms

Histograms are visual tools to examine distributions. A histogram is a bar graph that shows how frequently data fall within specific *cells*, that is, ranges of values. Histograms make it relatively simple to estimate where the process is centered and how much fluctuation there is about the center.

Table 1 (Page 4) shows measurements of widths (in inches) of 125 wood components produced by a woodworking machine.

We must have data to know how a process is performing. However, it is difficult to derive much information from data as presented in Table 1. The data would provide more information if they were grouped, organized, and displayed graphically. A histogram does just that.

We will leave the detailed discussion of how to create histograms for a future publication. For our purposes here, we will

simply state that creating a histogram involves developing and plotting a *frequency distribution*. A frequency distribution is a tally of measurements within specific cells. For example, in Table 1 there are 15 measurements in the 2.5395–2.5404 cell, 21 measurements in the 2.5405–2.5414 cell, and so on. The frequency distribution is shown in Table 2, and the histogram is shown in Figure 2.

What does this histogram tell us? First, we can easily estimate where the process is centered and the amount of spread about the center. The center of this distribution is approximately 2.542 inches. We can see that the majority of the measurements are between 2.540 and 2.544 inches.

Knowing the specifications enables us to get more information from the histogram. If, for example, we knew the specifications were 2.542 inches \pm 0.008 inch, we would say the process was performing quite well. If, on the other hand, the specifications were 2.544 inches \pm 0.002 inch, the histogram shows that a substantial amount of material is being produced below the lower specification and thus is defective.

A histogram is a snapshot of the process. It is useful for examining the status (centering and spread) of the process at the time the data were collected and for examining the general *shape* (for example, number of peaks and symmetry) of the distribution.

A single histogram, however, does not allow us to evaluate process performance through time nor does it allow us to determine whether the process was stable (consistent and predictable

Table 1.—Sample data.

2.541	2.542	2.541	2.542	2.543	2.544	2.539	2.542	2.545	2.543	2.543
2.540	2.544	2.539	2.540	2.542	2.540	2.545	2.543	2.540	2.543	2.541
2.542	2.542	2.541	2.543	2.543	2.543	2.543	2.543	2.543	2.540	2.544
2.541	2.541	2.544	2.544	2.544	2.543	2.542	2.546	2.542	2.542	2.543
2.543	2.537	2.542	2.541	2.540	2.542	2.542	2.542	2.543	2.544	2.541
2.543	2.538	2.543	2.541	2.541	2.541	2.545	2.545	2.543	2.543	
2.541	2.540	2.542	2.542	2.541	2.541	2.542	2.540	2.542	2.540	
2.541	2.541	2.544	2.543	2.543	2.541	2.542	2.542	2.542	2.539	
2.543	2.541	2.543	2.540	2.543	2.542	2.544	2.540	2.542	2.542	
2.543	2.543	2.543	2.543	2.543	2.542	2.545	2.541	2.540	2.542	
2.541	2.542	2.543	2.540	2.542	2.543	2.539	2.542	2.543	2.542	
2.541	2.542	2.540	2.540	2.542	2.542	2.542	2.542	2.544	2.542	

Table 2.—Frequency distribution for data in Table 1.

Cell boundaries	Cell midpoint	Frequency
2.5365 – 2.5374	2.537	1
2.5375 – 2.5384	2.538	1
2.5385 – 2.5394	2.539	4
2.5395 – 2.5404	2.540	15
2.5405 – 2.5414	2.541	21
2.5415 – 2.5424	2.542	35
2.5425 – 2.5434	2.543	32
2.5435 – 2.5444	2.544	10
2.5455 – 5.5454	2.545	5
2.5455 – 2.5464	2.546	1

A histogram. . .

is a useful snapshot of the process, but it doesn't let us evaluate the process over time or determine whether the process is stable.

performance) when the data were collected. Also, it takes time to collect the data (mathematicians suggest at least 50 to 100 data points per histogram) which can become overwhelming if it has to be done every day.

To be practical for day-to-day process control, we need a system in which relatively small samples allow us to decide whether the process is okay and therefore should be left alone, or whether problems are beginning to arise and we should take action. SPC provides such a system through the use of *control charts*.

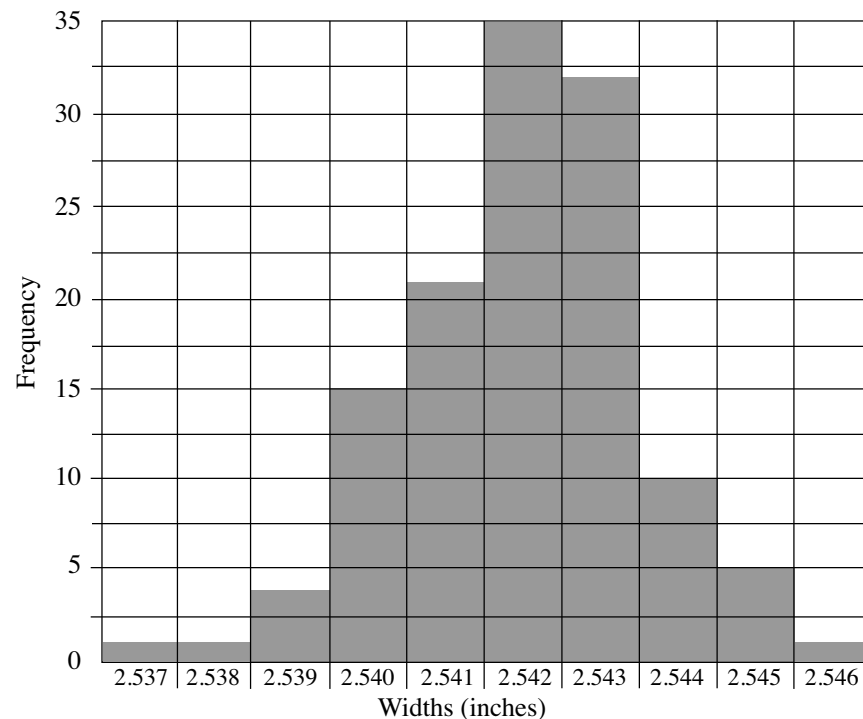


Figure 2.—Histogram for data in Table 1.

Control charts

Control charts are an SPC tool used to monitor and control processes. There are charts for *variables data* (measurement data such as length, width, thickness, and moisture content) and charts for *attributes data* (“counts” data such as number of defective units in a sample or number of errors on an invoice). We’ll focus here on one type of variables control chart and will discuss the other kinds in future publications.

Control limits...

tell us how far we can expect sample values to stray from the average, given the inherent variability of the process.

In general, control charts are used as follows: samples are taken from the process, statistics (for example, average and range) are calculated and plotted on charts, and the results are interpreted with respect to *process limits*—or, as they are known in SPC terminology, *control limits*. Control limits are the limits within which the process operates under normal conditions. They tell us how far we can expect sample values to stray from the average given the inherent variability of the process—or, to use the SPC terms, the *magnitude of common-cause variation*. Data points beyond the control limits or other unusual patterns indicate special-cause variation.

Estimating control limits

Calculating control limits requires only two numbers: an estimate of *central tendency* (process centering), and an estimate of *process variation*.

The *average* is our best estimate of central tendency. The average is widely used and is understandable to most people. For example, golfers and bowlers routinely calculate averages from a list of scores. To find an average, add all the sample measurements and divide the sum by the sample size. As an example, let’s use the first five sample measurements in Table 1:

$$\begin{aligned}2.541 + 2.540 + 2.542 + 2.541 + 2.543 &= 12.707 \\ \bar{x} &= 12.707 \div 5 \\ \bar{x} &= 2.541 \text{ (rounded to three decimal places)}\end{aligned}$$

where \bar{x} (pronounced “X-bar”) is the symbol used for the average.

In addition to a measure of central tendency, we need a measure of variation. The values commonly used to quantify variation are the *standard deviation* and the *range*.

SPC uses the range more often than the standard deviation because calculating the standard deviation is fairly involved. Also, the standard deviation usually is a less familiar concept for most

people. The range (R), on the other hand, is simply the largest value (X_{\max}) in the sample minus the smallest value (X_{\min}) in the sample. For the five-sample measurements listed above, $X_{\max} = 2.543$ and $X_{\min} = 2.540$, and therefore:

$$R = 2.543 - 2.540 = 0.003$$

We now have the two values we need to calculate the control limits—a measure of central tendency (the average, \bar{x}) and a measure of variation (the range, R). Recall that control limits tell us how far we can expect sample values to stray from the average, given the magnitude of process variation. Therefore, the formula for the control limits must account for both the average and the range. A frequency distribution allows us to develop such a formula.

Frequency distributions and probability

Recall that Table 2 was the frequency distribution for the data in Table 1. The science of statistics provides us with a number of frequency distributions with known *probabilities*. In common language, probabilities quantify the odds or likelihood of a specific event. For example, the probability of getting the ace of spades in a single draw from a deck of cards is $1/52$ (1 of 52 possible outcomes), or approximately 0.019. The probability of winning the lottery is often on the order of 1 in 1,000,000 (0.000001). As you can see, the smaller the probability, the less likely the event.

Statistics science also enables us to determine the probability that something will **not** happen. For example, the probability of not getting the ace of spades in a single draw from a deck of cards is 1 minus the fraction that indicates the probability of getting the ace of spades, which is 1 minus $1/52$ (or 0.019), which is 0.981. For more information on probability, see OSU Extension publication EM 8718, *An Introduction to Models and Probability Concepts*.

In SPC, we use known frequency distributions to establish control limits with known probabilities. Control limits are established so that the probability of obtaining a value (that is, a result) beyond the limits is very small unless the process changes significantly. Therefore, control limits minimize false alarms—that is, searching for problems when none exists. Searching for problems is often expensive because it involves time, effort, and, in many cases, equipment downtime. Minimizing false alarms and their expense is a key benefit of SPC and is the prime reason the first book on SPC was titled “Economic Control of Quality of Manufactured Product” (Shewhart, 1931).

The normal distribution

The *normal* distribution* is a frequency distribution used extensively in SPC. The normal distribution commonly is called the *bell curve* due to the curve's shape (Figure 3). Only two parameters are needed to construct a normal distribution: the average and the standard deviation. The average is designated with the Greek lowercase letter μ (mu), and the standard deviation is designated with the Greek lowercase letter σ (sigma). The normal curve's peak is at the average, μ , and its *spread* (the width of the curve) is reflected by the standard deviation, σ . The parameters μ and σ generally are unknown, and thus we have to estimate them by sampling the process. We use \bar{x} , the sample average, to estimate μ , and we use R , the sample range, to estimate σ .

In SPC, the normal distribution allows us to determine the probabilities of getting values beyond control limits. Probability values for the normal distribution can be found in textbooks on statistics and quality control. For purposes of our brief discussion of normal distribution, we simply will state some common probabilities.

The probability that a normally distributed variable will be within plus or minus 1 standard deviation (1σ , pronounced “one

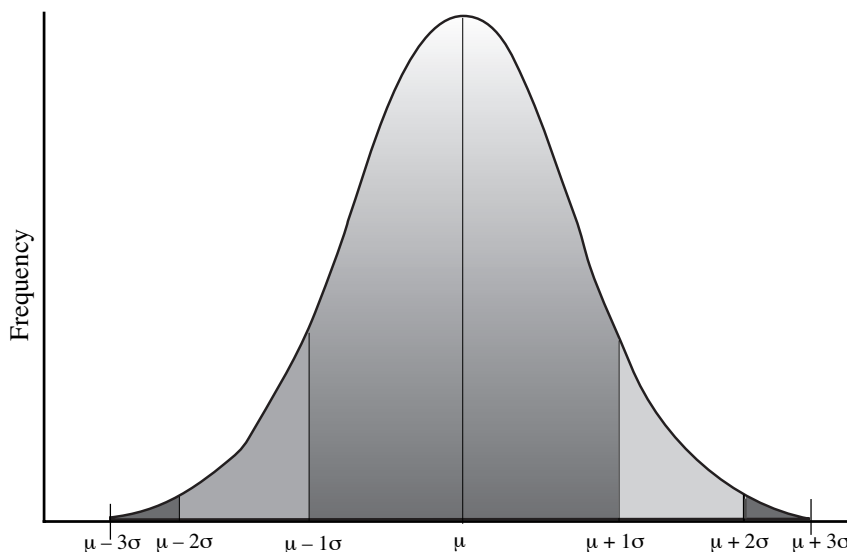


Figure 3.—The normal distribution.

*Note: “Normal” is the name for the distribution and is not to be confused with common usage of the word “normal” as meaning “standard” or “commonplace.”

sigma”) of the average is approximately 0.68. That is, 68 percent of the values will be within 1 standard deviation of the average. The probability that a normally distributed variable will be within $\pm 2\sigma$ of the average is approximately 0.95, and within $\pm 3\sigma$ of the average is approximately 0.997 (Figure 4).

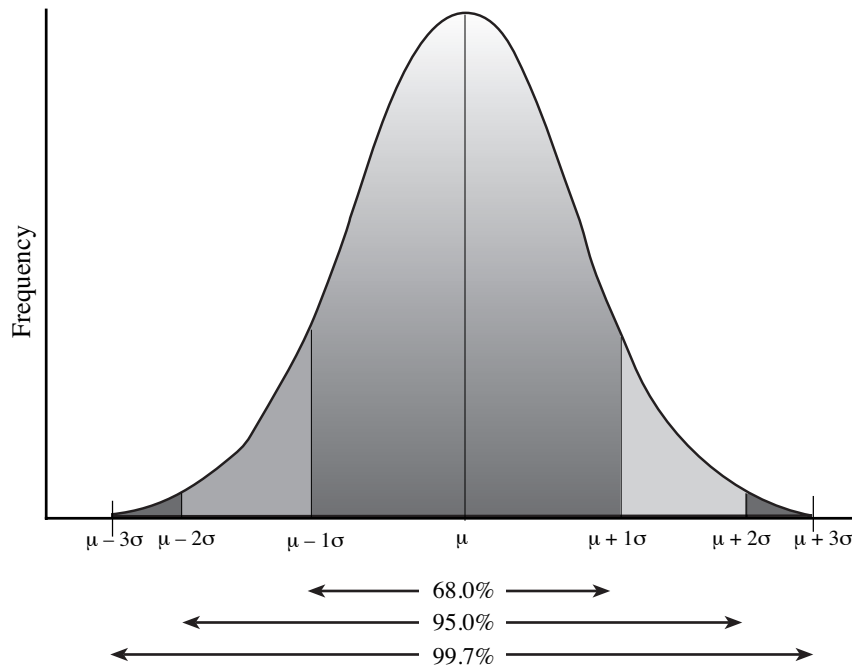


Figure 4.—The normal distribution, showing common probabilities.

Knowing that 99.7 percent of the values will fall within $\pm 3\sigma$ of the average, we can feel confident that a value beyond 3σ would be highly unlikely unless a significant change (i.e., special-cause variation) had occurred in the process.

For example, if process output is normally distributed, and the average for our process is 2.542, and the standard deviation (σ) is 0.002, then the probability of a value between 2.536 and 2.548 [that is, $2.542 \pm (3 \times 0.002)$] inches is 0.997. Conversely, the probability of observing a value less than 2.536 or greater than 2.548 is 0.003 ($1 - 0.997$), or approximately 3 chances in 1,000. Therefore, 2.536 and 2.548 would appear to make good control limits because a value beyond the limits is statistically rare. Therefore, it is very likely that special causes of variation are influencing the process, and so searching for problems is likely to be profitable.

So, can we use 2.536 as the lower control limit and 2.548 for the upper control limit? We could—if we were examining **individual** items from the process. Recall, however, that we are sampling the

process and collecting statistics (in this example, the average) for multiple items* rather than for individual items. Our control chart therefore should reflect the distribution of **averages**, not of individual data points.

Individuals vs. averages

The variation of averages is significantly less than the variation of the individual values. Figure 5 demonstrates this point by overlaying a histogram of averages of five measurements from the data in Table 1 onto the histogram of individual values.

The larger the sample, the narrower the distribution of sample averages. Therefore, control limits for averages will be *narrower* (closer to the average) than control limits for individual measurements. The values needed to calculate control limits have been tabulated and are available in textbooks. To calculate control limits, you need only look up in a table the value that corresponds to sample size, and then perform simple multiplication and addition.

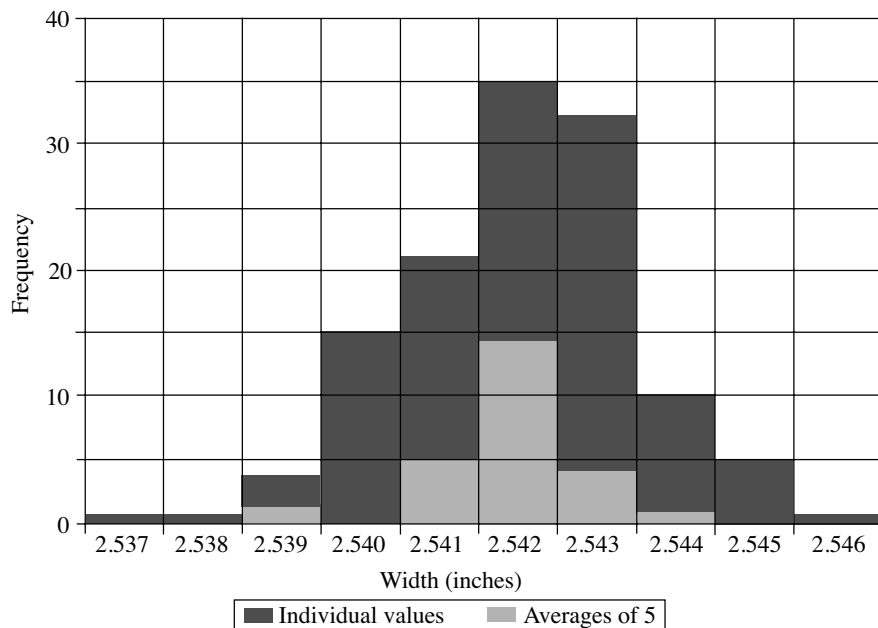


Figure 5.—Histogram for individual values and for averages of 5 measurements.

*There are several reasons we prefer to sample multiple items versus individual items from the process. One reason is that the Central Limit Theorem allows us to use the normal distribution regardless of the actual distribution of the process. We will discuss this and the other reasons in detail in future publications.

Control limits vs. specification limits

Before leaving the issue of control limits, we must address the often-asked question, “Why can’t we simply use the specifications for control limits? Why all this bother with control limits based on statistical probabilities, 3σ , etc. when all we really want to know is how much ‘good’ (that is, ‘within the specifications’) product we’re producing?” There are two primary reasons **never** to use specification limits as control limits.

Control limits are established to minimize false alarms. Unless some significant change has occurred in the process, a sample value beyond a 3σ control limit is a statistical rarity and thus a signal that special-cause variation is influencing the process. Searching for problems is likely to be profitable. Therefore, control limits must be established as a function of the capabilities of the process. In practice, specification limits usually are established by engineers and are not a function of the capabilities of the process. Control limits represent “what the process **can** do,” and specification limits represent “what we **want** the process to do.” To be useful for quality control, limits must be based on what the process can do.

Another reason never to use specification limits as control limits is that the former are for individual items, not for averages. As noted earlier, the variation for averages is less than the variation for individual values. Therefore, comparing a sample average to a specification limit is the proverbial apples-to-oranges comparison.

False alarms (chasing problems that aren’t there) and failing to detect problems are common when specifications instead of control limits are used on control charts.

Control limits ...

represent “what the process *can* do.”
Specification limits represent “what we *want* the process to do.”

Determine whether the process is stable

How can we be sure that the data we use to establish control limits are not simply a snapshot of an unstable process? If the process is not stable, we are shooting at a moving target, and our control limits are meaningless for long-term process control. Therefore, we must find a way to determine whether the process is stable. In SPC terminology, we ask, “Is the process in statistical control?” Or, more simply, “Is it *in control*?”

In common usage, the phrase “in control” generally describes a desirable situation, and it has the same connotation in SPC. On the other hand, the phrase “out of control” brings forth images of mayhem—machines on fire, nuts and bolts flying through the air, etc. An out-of-control process, as the phrase is used in SPC, is not quite so dramatic.

Deming (1982) defines control in SPC:

A stable process, one with no indication of a special cause of variation, is said to be, following Shewhart, in statistical control, or stable. It is a random process. Its behavior in the near future is predictable.

Therefore, *out of control* can be defined as a process that is under the influence of special causes of variation. Its behavior is not predictable. **Statistically rare occurrences are signals that a process is out of control.** When a process is out of control, we should investigate to find and eliminate the special causes of variation in order to bring the process back in control and thereby improve consistency of product quality.

In our discussion of the normal distribution, above, we stated that upper and lower control limits in SPC generally were set at plus or minus 3 standard deviations ($\pm 3\sigma$) from the average. Finding a normally distributed value beyond the 3σ limits has a probability of approximately 0.003, which means it is statistically rare. Therefore, we can state that a process is out of control if a sample value falls outside the control limits.

There are other rules in SPC for determining whether a process is in or out of control. Some companies simply use the “outside the control limits” rule just mentioned; others use several rules that involve detecting trends or “runs” of data points above and below average. For example, eight consecutive data points on the chart on the same side of the center line (the average) or six consecutive points on the chart steadily increasing or decreasing may indicate a change in the process and therefore an out-of-control situation. These additional rules often help to detect problems sooner than waiting for a point to fall outside the control limits. For this discussion, we will focus simply on the outside-the-control-limits rule. Other rules will be discussed in a future publication.

Now, we have most of the information we need to determine whether the process is in control. However, for practical application, one thing is lacking: a visual tool to help us evaluate sample data and compare them to the control limits. For this purpose, Shewhart created the control chart, also known as the Shewhart Control Chart in honor of its inventor.

Control charts provide a graphical view of the process over time. We will use the data in Table 1 to construct a control chart for the average, known as an \bar{x} chart. For purposes of the example, we group the measurements in Table 1 into 25 samples with 5 measurements per sample. The grouped data are shown in Table 3.

Table 3. Data from Table 1 organized in samples of 5.

Sample no.	Items in sample*					Summary of sample	
	1	2	3	4	5	\bar{x}	R
1	2.541	2.540	2.542	2.541	2.543	2.541	0.003
2	2.543	2.541	2.541	2.543	2.543	2.542	0.002
3	2.541	2.541	2.542	2.544	2.542	2.542	0.003
4	2.541	2.537	2.538	2.540	2.541	2.539	0.004
5	2.541	2.543	2.542	2.542	2.541	2.542	0.002
6	2.539	2.541	2.544	2.542	2.543	2.542	0.005
7	2.542	2.544	2.543	2.543	2.543	2.543	0.002
8	2.540	2.542	2.540	2.543	2.544	2.542	0.004
9	2.541	2.541	2.542	2.543	2.540	2.541	0.003
10	2.543	2.540	2.540	2.543	2.542	2.542	0.003
11	2.543	2.544	2.540	2.541	2.541	2.542	0.004
12	2.543	2.543	2.543	2.542	2.542	2.543	0.001
13	2.544	2.540	2.543	2.543	2.542	2.542	0.004
14	2.541	2.541	2.541	2.542	2.542	2.541	0.001
15	2.543	2.542	2.539	2.545	2.543	2.542	0.006
16	2.542	2.542	2.545	2.542	2.542	2.543	0.003
17	2.544	2.545	2.539	2.542	2.542	2.542	0.006
18	2.543	2.543	2.546	2.542	2.545	2.544	0.004
19	2.540	2.542	2.540	2.541	2.542	2.541	0.002
20	2.542	2.545	2.540	2.543	2.542	2.542	0.005
21	2.543	2.543	2.542	2.542	2.542	2.542	0.001
22	2.540	2.543	2.544	2.543	2.543	2.543	0.004
23	2.540	2.542	2.544	2.543	2.540	2.542	0.004
24	2.539	2.542	2.542	2.542	2.542	2.541	0.003
25	2.543	2.541	2.544	2.543	2.541	2.542	0.003
						Averages for all samples	
						$\bar{\bar{x}}$	\bar{R}
						2.542	0.003

*In practice, measurements should be grouped in a manner Shewhart called “rational subgrouping.” Sampling should be done so that differences between subgroups (samples) are maximized and differences within subgroups are minimized. We will discuss this subject in depth in a future publication.

To construct the control chart, we first calculate the average and range for each sample. We then calculate $\bar{\bar{x}}$ (pronounced “x-bar-bar” or “x double bar”), which is the average of the 25 sample averages, and we calculate \bar{R} , which is the average of the 25 sample ranges.

The grand average, $\bar{\bar{x}}$, for the 25 sample averages is 2.542 inches. We draw a horizontal line across the center of the graph at 2.542 to represent the average. \bar{R} is 0.003 inch. As discussed previously, to calculate the upper and lower control limits (UCL and LCL, respectively) we use formulas and table values from textbooks. Using Table D in Appendix VI in Montgomery (1997), the table value (A_2) for a sample size of 5 is 0.577. The control limits are then:

$$\begin{aligned} \text{UCL} &= \bar{\bar{x}} + A_2 \bar{R} \\ &= 2.542 + (0.577 \times 0.003) \\ &= 2.544 \end{aligned}$$

$$\begin{aligned} \text{LCL} &= \bar{\bar{x}} - A_2 \bar{R} \\ &= 2.542 - (0.577 \times 0.003) \\ &= 2.540 \end{aligned}$$

We now draw horizontal lines on the graph at the appropriate locations for the UCL and LCL. Last, data points for each sample average are plotted on the chart (Figure 6).

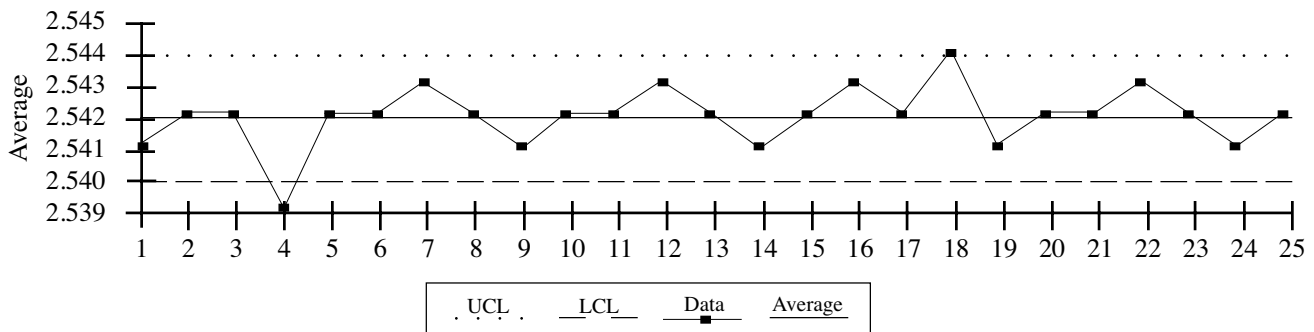


Figure 6.— $\bar{\bar{x}}$ control chart for data in Table 3.

Given that we are attempting to determine whether the process is in or out of control, what can we learn from this chart? Recall that a point outside the control limits or some other unusual patterns indicate the process is out of control. Therefore, Sample 4 indicates the process is out of control, and Sample 18 is questionable

because it is on the upper control limit.* Had these data been collected as the process was operating, we would have looked for a source of trouble immediately after we plotted the average of Sample 4. Because the data are historical, we are using the data only to determine whether the process is in control.

The chart indicates the process was out of control when the data were collected. Actions should be taken to identify the problems that led to the low value in Sample 4 and the high value in Sample 18. After these problems have been identified and corrected, we remove out-of-control samples from the data and recalculate $\bar{\bar{x}}$, \bar{R} , and the control limits. We then redraw the charts and check to see whether the process is in control now that the out-of-control data points have been removed. We continue this process until all data points are in control, and then use the resulting limits as trial limits for future production.

Had the initial control chart shown that the process was in control, the initial control limits would serve as trial control limits for future production.

Continue to monitor and control the process

For day-to-day process monitoring, we will collect samples and plot the results on a control chart using the trial center line and control limits developed above. A data point beyond the limits and unusual patterns will continue to be used as indicators that some aspect of the process has changed and, therefore, as signals that we must search for special causes of variation. If special causes are found, corrective actions must be taken.

Due to the hectic pace of the manufacturing environment, it is common for long periods to elapse between sample collection and

*In practice, the control chart for variation, the R chart, always should be constructed and interpreted **first** to determine whether the process is in or out of control. However, we have chosen to sacrifice some technical accuracy in order to reduce the complexity of discussion about multiple control charts. We chose to discuss the \bar{x} chart here rather than the R chart because \bar{x} charts usually are easier for newcomers to SPC to understand. An \bar{x} chart monitors the fluctuation in the process over time—a pretty straightforward concept. An R chart, on the other hand, monitors the fluctuation in process **variability** over time; in other words, it monitors the variability of the variability.

plotting points on control charts. If an out-of-control situation is indicated, it is likely that defective product has been produced in the time between problem occurrence and problem detection; the longer this period, the more defective product has been produced. For control charts to be effective, data should be analyzed, plotted, and interpreted and actions taken as soon as possible. Therefore, control charts should be constructed and interpreted in real time by the workers on the mill floor, not in the office by managers.

Finally, because control limits are a function of process variation, they should be evaluated periodically and revised when control charts provide evidence that process variation has been reduced.

Compare process performance to specifications: Process capability analysis

Up to this point, we have concentrated on monitoring what the process **is** doing but have shown little consideration for what we **want** the process to do. In other words, we have paid little attention to how the process conforms to specifications.

The fact is, an in-control process can produce defective product if the process is off-target or if the common-cause variation is too high. As stated above, the first steps in an SPC program should be to establish control. An out-of-control process is unstable, and therefore estimates of process performance (centering and variation) are of little use. Only after establishing control can we examine the process's ability to meet specifications.

Let's begin with a graphical look at process variation.

Figure 7 shows normal curves for two process distributions. LSL and USL are the lower specification limit and upper specification limit, respectively. Both processes are centered on the target dimension. The lower, wider distribution represents a process with relatively high variation (high standard deviation); the taller, narrower distribution represents a process with lower variation (low standard deviation). The shaded area represents material outside specifications.

The advantages of reducing variation are obvious. The process with lower standard deviation produces far less defective material. From a practical standpoint, however, we need to **quantify** the relationship between the spread of the process and the spread of the specifications. A process-capability analysis does this by using capability indices.

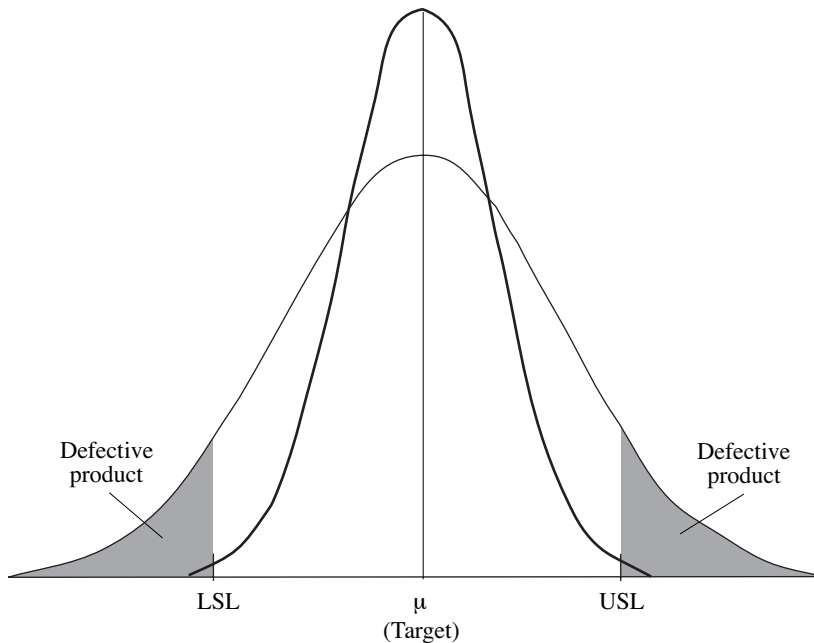


Figure 7.—Effect of process standard deviation on percentage of product that does not meet specifications.

The first capability index we will discuss is C_p which is calculated as:

$$C_p = (USL - LSL) \div 6 \hat{\sigma}$$

where σ is the process standard deviation and the “^” (pronounced “hat”) symbol over it means “estimate.”

Recall that σ is the **true** standard deviation for a normally distributed variable, and generally the best we can do is to estimate it by sampling the process. The value $6 \hat{\sigma}$ (plus and minus 3σ) is the total width of process variation. Therefore, C_p is a ratio of the specification width to total process width.

Continuing the example using the data in Table 1, let’s say our specifications are 2.543 inches \pm 0.003 inch. Therefore, our LSL is 2.540 and our USL is 2.546 inches. To estimate standard deviation ($\hat{\sigma}$) we divide \bar{R} (the average range) by d_2 , a table value found in SPC textbooks. Appendix VI in Montgomery (1997) lists d_2 as 2.326 for samples of size 5 (i.e., samples of five measurements). Recall our estimate of \bar{R} is 0.003; therefore, $\hat{\sigma}$ is $0.003 \div 2.326$, or 0.001 inch. C_p is then:

$$C_p = (2.546 - 2.540) \div (6 \times 0.001) = 1.0$$

What does this mean? In simple terms, C_p less than 1.0 is “bad” and greater than 1.0 is “good.” That is because a C_p less than 1.0

***C_p doesn't
account for...***

process centering. In theory, a process could turn out 100% defective product, yet if process variability were low, C_p would be okay.

indicates that process variation is higher than the specification width, and therefore too much material is defective. A C_p greater than 1.0 indicates that process variation is less than the specification width, and therefore the process can meet the specifications while producing minimal defects. Because our C_p is 1.0, we know that the process variation is equal to the specification width, and therefore the process is (just barely) capable of meeting the specifications.

C_p does not account for process centering relative to the target. (Theoretically, a manufacturing process could be centered far away from the target specification and therefore producing 100% defective product; yet if the process variability were low, C_p would indicate everything was okay.)

To account for process variation **and** for centering relative to the target, we use another process capability index, C_{pk}. The formula for C_{pk} is:

$$C_{pl} = (\hat{\mu} - LSL) \div 3\hat{\sigma}, \quad C_{pu} = (USL - \hat{\mu}) \div 3\hat{\sigma}$$

$$C_{pk} = \min\{C_{pl}, C_{pu}\}$$

where C_{pl} and C_{pu} are the lower and upper process capability indices, respectively, relative to the process center; $\hat{\mu}$ is our estimate of process centering; and “min” indicates that C_{pk} is the minimum (lesser) of C_{pl} and C_{pu}.

To calculate C_{pk}, we use $\hat{\sigma}$ as calculated above, and we use $\bar{\bar{x}}$ as $\hat{\mu}$. (Recall that, previously, $\bar{\bar{x}}$ was calculated as 2.542.) C_{pk} is then:

$$C_{pl} = (2.542 - 2.540) \div (3 \times 0.001) = 0.667,$$

$$C_{pu} = (2.546 - 2.542) \div (3 \times 0.001) = 1.333$$

$$C_{pk} = \min\{0.667, 1.333\} = 0.667$$

C_{pk} is interpreted much the same as C_p; that is, below 1.0 is “bad” and above 1.0 is “good.” We get a bit more information with C_{pk}, however. Because C_{pl} and C_{pu} are not equal, we know our process is off-center. More specifically, because C_{pl} is lower than C_{pu}, we know we are centered too close to the lower specification limit. Therefore, excessive defects are produced below the lower specification limit. Furthermore, using the normal distribution as an approximation, we can calculate that about 2.3 percent (about 2.3 defects per 100) of the material will be defective (out of spec). Calculations of this type will be discussed in a later publication.

What if we adjusted the process to put it on target; that is, we shifted the process center ($\bar{\bar{x}}$) from 2.542 to 2.543 inches? C_p does not account for process centering, and so it would not be affected. C_{pk} , however, would increase from 0.667 to 1.0, meaning that our defect rate would decrease from about 2.3 percent to approximately 0.27 percent (about 2.7 defects per 1,000), a very significant reduction.

In addition to adjusting the process to put it on target, what if we could reduce process variation from 0.001 to 0.0008 inch? This would result in C_p and C_{pk} of 1.25. The defect rate would drop to 0.009 percent (approximately 9 defects per 100,000). This is obviously a very significant improvement in quality. After obtaining cost estimates for scrap and rework, we can calculate the effect that percentage reductions in defects will have on profit increases and thus determine the benefits of quality improvement to the bottom line.

The profit increases due to decreases in variation are substantial. The question becomes, how do we reduce variation? The answer: through continuous process improvement.

Continuous process improvement

As stated previously, real quality improvement requires a continual focus on reducing common-cause variation. Reducing common-cause variation is possible only after the process has been brought into control.

Process improvement (which leads to quality improvement) requires a more systematic and structured approach than usually required to remove special causes of variation. Companywide quality improvement requires:

- Determining customer needs
- Setting quality goals
- Developing an improvement strategy
- Providing the necessary training and resources
- Establishing the organizational infrastructure (quality councils, teams, etc.)
- Reviewing progress, and
- Revising the reward system

For these reasons, quality improvement requires the commitment and involvement of upper management. Years of experience have shown that “delegating quality” is ineffective. Juran (1989)

provides a thorough treatment of management's role in quality planning, control, and improvement.

Conclusions

The bottom line is: SPC helps manufacturers increase their competitiveness and profitability. We have demonstrated SPC's foundation in mathematics and statistics to build your understanding and confidence in SPC as a profit-making tool and to overcome any preconceptions that SPC is yet another management fad.

We hope this publication has convinced you that implementing SPC will make your company money, and we hope you will commit to training your personnel to use SPC. To begin the training process, we refer you to Part 3 in this series, *Starting an SPC Program*.

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Glossary

Assignable causes of variation—See *special causes of variation*.

Attributes data—Qualitative data that can be counted for recording and analysis. Results may be recorded as yes/no, go/no go, or defective/not defective. Examples include percent defective in a *sample* and number of blemishes on a surface.

Arithmetic mean—See *average*.

Average—A measure of location or *central tendency* which is the sum of the observed values divided by the number of observations. Also called the *arithmetic mean* or, simply, the *mean*.

Bell curve—Common name for the *normal distribution*, a name derived from the shape of the curve.

Cell—A grouping of values between specified upper and lower boundaries used to create *frequency distributions*.

Center (centered, centering)—A numerical value that is “typical” for a set of data. Values used include the *average*, the *median*, and the *mode*.

Central tendency—See *center*.

Chance causes of variation—See *common causes of variation*.

Common causes of variation—Sources of variation that affect all the individual values of the process output being studied. The sources generally are numerous and individually of small importance but cannot be detected or identified. Also called *chance*, *random*, and *unknown causes* of variation.

Control (statistical)—The condition that exists after a process in which all *special causes* of variation have been eliminated and only *common causes* remain.

Control chart—A graphic representation of a characteristic of a process, showing plotted values of some statistic gathered from the characteristic, a central line, and one or two *control limits*. Used to determine whether a process is in *statistical control* and to help maintain statistical control.

Control limits—On a *control chart*, the criteria for signaling the need for action, or for judging whether a set of data does or does not indicate a “state of *statistical control*.” Control limits are calculated from process data and are not to be confused with *specification limits*.

Distribution—See *frequency distribution*.

Frequency distribution—A tally of the count, or frequency, of occurrences of data in specific *cells*.

Histogram—A bar chart for displaying a *frequency distribution*.

In control—See *control (statistical)*.

Mean—See *average*.

Median—The value at the midpoint in the ordered range of values: half the values are greater than the median value, and half the values are less than the median value.

Mode—The most frequently observed value.

Normal distribution—A continuous, symmetrical, bell-shaped *frequency distribution* for variables data that underlies *control charts* for variables.

Out of control—The absence of conditions described in *control (statistical)*.

Probability—A scientific discipline whose objective is to study uncertainty. Probability is the likelihood (commonly called the “odds”) that a specific event will occur.

Process limits—See *control limits*.

Random causes of variation—See *common causes of variation*.

Range—A measure of dispersion; the difference between the largest observed value and the smallest observed value in a given *sample*.

Sample—A group of items, observations, test results, or portions of material taken randomly from a larger collection of items, observations, test results or quantities of material, which provide information that may be used as a basis for making a decision about the larger collection. See also *subgroup*.

Special causes of variation—Sources of variation that are intermittent, unpredictable, and unstable and that can be detected and identified.

Specification limits—The engineering requirement for judging acceptability of a particular characteristic. Specifications are not to be confused with *control limits*.

Spread—General term describing the dispersion or variability in a data set. Commonly measured with the *range* or *standard deviation*.

Standard deviation (sample)—A measure of dispersion, calculated as the square root of the sum of the squared deviations of observations from their average divided by one less than the number of observations. The *range* often is used to estimate the standard deviation.

Subgroup—In process control applications, generally synonymous with *sample*.

Unknown causes of variation—See *common causes of variation*.

Variables data—Quantitative data, where measurements are used for analysis. Examples include length, width, thickness, viscosity, strength (e.g., pounds per square inch, or psi), and density.



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