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Using the Simplex Method to Solve Linear Programming Maximization Problems

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A key problem faced by managers is how to allocate scarce resources among activities or projects. Linear programming, or LP, is a method of allocating resources in an optimal way. It is one of the most widely used operations research (OR) tools. It has been used successfully as a decision-making aid in almost all industries, and in financial and service organizations.

Programming refers to mathematical programming. In this context, it refers to a planning process that allocates resources—labor, materials, machines, and capital—in the best possible (optimal) way so that costs are minimized or profits are maximized. In LP, these resources are known as *decision variables*. The criterion for selecting the best values of the decision variables (e.g., to maximize profits or minimize costs) is known as the *objective function*. The limitations on resource availability form what is known as a *constraint set*.

For example, let's say a furniture manufacturer produces wooden tables and chairs. Unit profit for tables is $6, and unit profit for chairs is $8. To simplify our discussion, let's assume the only two resources the company uses to produce tables and chairs are wood (board feet) and labor (hours). It takes 30 bf and

About this series

According to the Operations Research Society of America, "Operations research [OR] is concerned with scientifically deciding how to best design and operate man-machine systems, usually under conditions requiring the allocation of scarce resources."

This publication, part of a series, should be useful for supervisors, lead people, middle managers, and anyone who has planning responsibility for either a single manufacturing facility or for corporate planning over multiple facilities. Although managers and planners in other industries can learn about OR techniques through this series, practical examples are geared toward the wood products industry.

See page 21 for information about additional publications in this series.

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Decision variables. . .

"The resources available."

Constraint set. . .

"The limitations on resource availability."

Objective function. . .

"The criterion for selecting the best values of the decision variables."

5 hours to make a table, and 20 bf and 10 hours to make a chair. There are 300 bf of wood available and 110 hours of labor available. The company wishes to maximize profit, so profit maximization becomes the *objective function*. The resources (wood and labor) are the *decision variables*. The limitations on resource availability (300 bf of wood and 110 hours of labor) form the *constraint set,* or operating rules that govern the process. Using LP, management can decide how to allocate the limited resources to maximize profits.

The "linear" part of the name refers to the following:

- The objective function (i.e., maximization or minimization) can be described by a linear function of the decision variables, that is, a mathematical function involving only the first powers of the variables with no cross products. For example, $23X_2$ and $4X_{16}$ are valid decision variable terms, while $23X_2^2$, $4X_{16}^3$, and $(4X_1 * 2X_1)$ are not. The entire problem can be expressed as straight lines, planes, or similar geometrical figures.
- The constraint set can be expressed as a set of linear equations.

In addition to the linear requirements, nonnegativity conditions state that the variables cannot assume negative values. It is not possible to have negative resources. Without these conditions, it would be mathematically possible to use more resources than are available.

In EM 8719, *Using the Graphical Method to Solve Linear Programs*, we use the graphical method to solve an LP problem involving resource allocation and profit maximization for a furniture manufacturer. In that example, there were only two variables (wood and labor), which made it possible to solve the problem graphically.

Problems with three variables also can be graphed, but three-dimensional graphs quickly become cumbersome. Problems with more than three variables cannot be graphed. Most real-world problems contain numerous objective criteria and resources, so they're too complicated to represent with only two or three variables. Thus, for all practical purposes, the graphical method for solving LP problems is used only to help students better understand how other LP solution procedures work.

This publication will build on the example of the furniture company by introducing a way to solve a more complex LP problem. The method we will use is the *simplex* method.